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Comment on "Quantum Phase of Induced Dipoles Moving in a Magnetic Field"

by
C. R. Hagen
Department of Physics & Astronomy
University of Rochester
Rochester, NY 14627

Abstract

It has recently been suggested that an Aharonov-Bohm phase should be capable of detection using beams of neutral polarizable particles. A more careful analysis of the proposed experiment suffices to show, however, that it cannot be performed regardless of the strength of the external electric and magnetic fields.

It has recently been suggested [1] that an Aharonov-Bohm phase should be capable of detection using beams of neutral polarizable particles. A more careful analysis of the proposed experiment suffices to show, however, that it cannot be performed regardless of the strength of the external electric and magnetic fields.

To demonstrate this result one begins with the Lagrangian of ref. 1

$$L = \frac{1}{2}M\mathbf{V}^2 + \frac{1}{2}\alpha(\mathbf{E} + \mathbf{V}\times\mathbf{B})^2 \tag{1}$$

where α is the (intrinsically positive) polarizability of the particle and M is its mass. The electric field $\mathbf{E}$ is taken to have a magnitude k/r in a radial direction in a plane perpendicular to the uniform magnetic field $\mathbf{B}$. Upon applying the canonical formalism one readily obtains from Eq. (1) the relevant Hamiltonian in the form

$$H = \frac{1}{2}(M + \alpha B^2)^{-1}(\mathbf{p} + \alpha B\bar{\mathbf{E}})^2 - \frac{1}{2}\alpha\mathbf{E}^2$$

where (as in ref. 1) motion has been restricted to the plane perpendicular to the magnetic field. The two dimensional vector $\bar{\mathbf{E}}$ is the dual of $\mathbf{E}$ [i.e., $(\bar{\mathbf{E}})_i = \epsilon_{ij}E_j$].

One readily finds that the relevant Schrodinger equation for a particle of energy $\mathcal{E}$ is

$$\left[\frac{1}{2}\left(M+\alpha B^2\right)^{-1}\left(-i\hbar\boldsymbol{\nabla}+\alpha B\bar{\mathbf{E}}\right)^2-\frac{1}{2}\alpha\mathbf{E}^2\right]\psi=\mathcal{E}\psi.$$

Standard separation of variables then yields for the radial wave function $f_m(r)$ the result

$$\left[\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}+\frac{2(M+\alpha B^2)\mathcal{E}}{\hbar^2}-\frac{m^2+2m\alpha kB/\hbar-M\alpha k^2/\hbar^2}{r^2}\right]f_m(r)=0 \quad (2)$$

where $m=0,\pm1,\pm2,\ldots$.

The system described by Eq. (2) is one which allows quantum mechanically well-defined solutions only when the numerator of the $1/r^2$ term is positive for all m. This condition is readily seen to be violated in the case of $m=0$ whenever there is a nonvanishing electric field present. Moreover, the case $\mathbf{E}=0$ is clearly trivial in that it implies the absence of a quantum phase with the only effect of the interaction being a mass renormalization (i.e., $M\rightarrow M+\alpha B^2$).

In sum, the proposed experiment cannot be carried out because its assumptions are in basic conflict with quantum mechanics.

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References

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